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Practical Tests of Quantumness Using Multipartite Bell-Type Inequalities

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Abstract

As time has progressed, quantum processing units (QPUs) have become increasingly accessible, offering anyone the opportunity to run programs on real quantum hardware. However, the extent to which current QPUs demonstrate genuine quantum behavior still remains a vital question to address- after all it is important to know if real quantum computers are using quantum mechanics and not just expensive classical machines. To do this, the Bell inequalities can be used. Bell inequalities are mathematical tools used to detect entanglement and nonlocality; their violation confirms that the correlations produced by a QPU cannot be explained by any classical theory, allowing the confirmation of genuine quantum behavior. As a result, it is possible to use these tools to investigate the efficiency of Bell inequalities and their ability to withstand noise, in effect, allowing the evaluation of whether they make beneficial quantum tests. By generating three to five qubit GHZ states and applying optimized measurement settings designed to maximize inequality violation under both ideal and noisy conditions on a locally run simulator allows the graphing of how quickly Bell inequalities fall under the classical boundary based on the noise strength, conveying how easily they can test for quantumness despite noise. In conclusion, by evaluating multipartite Bell inequalities as a quantitative benchmark, this work contributes to the foundational understanding of testing how "quantum" today's quantum computers truly are. This study demonstrates that multipartite Bell inequalities provide a quantitative benchmark for assessing the genuine quantumness of contemporary quantum computers.

Keywords: quantum computing, Bell inequalities, quantum tests, quantum benchmarking

Introduction

Emerging quantum computing is expected to transform modern technology by utilising the principles of quantum mechanics to perform tasks that are either too complex or demanding for even some of the best classical computers. Recent developments in hardware have led to the development of highly advanced quantum processors, with the power to make quantum computation feasible (IBM Quantum, 2025; Quantinuum, 2024). Despite these rapid developments, the practicality and verifiable ability to perform non-classical computations of these systems remain an area of uncertainty. A major challenge is to ascertain genuinely non-classical behaviour in the context of noise and decoherence.

To address this, algorithms must do more than simply test individual qubits or hardware control units and check the ability of these computational devices to perform genuinely quantum operations. To create said benchmarks, the Bell Inequalities provide a solution. Bell Inequalities are mathematical tools used to test whether the correlations between measurement outcomes in a quantum system can be explained using classical physics—specifically, theories that assume particles have predefined properties. Hence, for a quantum computer to violate an inequality, it shows correlations stronger than classical theory therefore, indicating quantum behaviour.

In the early days of quantum mechanics, the violation of Bell inequalities stood as a profound testament to the nonlocal nature of reality. Bell's theorem revealed that no theory based on local hidden variables could reproduce the predictions of quantum mechanics. For decades, these violations were viewed primarily as philosophical curiosities - deep insights into the structure of nature, but with no clear path to technological utility. Today, that perception has changed.

One key example is communication complexity. One of the first practical domains where Bell inequality violations revealed a computational edge was communication complexity (Brukner et al., 2004). The central question here is: how much information must be exchanged between distant parties to solve a joint problem? In a pivotal 2002 paper, (Żukowski & Brukner, 2002) showed that whenever quantum strategies violate a Bell inequality, they can be harnessed to reduce the communication required between parties solving certain distributed tasks. This established that nonlocal correlations are a resource – they can save bits. Years later, (Buhrman et al., 2016) demonstrated the converse: any communication complexity advantage due to quantum strategies necessarily implies a Bell violation (Buhrman et al., 2010). This duality solidified the idea that Bell inequalities are not just about foundations—they are intimately tied to computational efficiency. Further refinements by researchers like Tavakoli have clarified when and how these advantages arise, suggesting a tight connection between fundamental quantum nonlocality and operational benefits in communication tasks (Tavakoli et al., 2020).

Another application of Bell inequalities is randomness certification. Random number generation is a cornerstone of secure computation, yet traditional generators are vulnerable to side-channel attacks and hardware imperfections. Quantum mechanics, by contrast, promises genuine randomness—and Bell inequalities provide the tool to certify it. In 2009, (Pironio et al., 2010) used Bell inequality violations in a loophole-free setting to generate and certify random bits, even when the devices involved could not be trusted. This was the first demonstration of device-independent randomness certification, where the very violation of the inequality proves that the outcomes cannot be predetermined or classically simulated (Pironio et al., 2010). More recent work, such as (Foreman et al., 2023) has expanded the scope of this idea. Today, protocols based on Bell tests run on quantum computers and real-world hardware, offering practical pathways to secure, private, and certifiable randomness for applications ranging from cryptography to simulations.

Bell inequalities also have cryptographic applications. The same principles that allow for randomness certification extend naturally to cryptography, where one of the most advanced frontiers is device-independent quantum key distribution (DIQKD). Here, two parties can generate a shared secret key even if they do not trust their hardware—so long as a Bell inequality is violated. Groundbreaking experiments like (Hensen et al., 2015) and follow-ups using superconducting qubits have demonstrated loophole-free Bell tests with increasing reliability

(Hensen et al., 2015). These experiments validate the feasibility of using Bell violations not only for foundational tests, but as the security backbone of next-generation cryptographic systems.

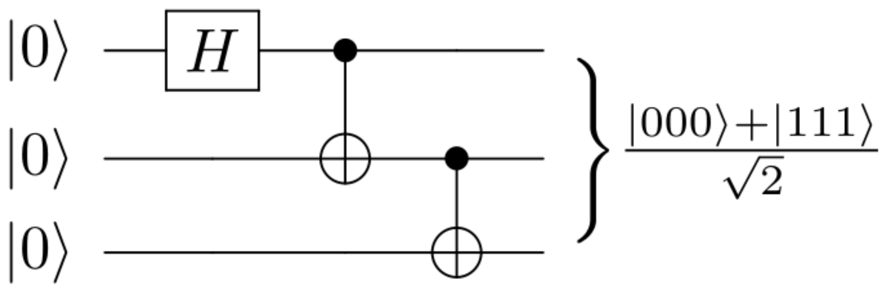
Finally, a key use of Bell inequalities is in quantum nonlocal games. In the realm of theoretical computer science, Bell-type experiments are being reimagined as nonlocal games—cooperative tasks where multiple players try to win based on shared strategies but without communication. These games are a playground for studying the separation between classical and quantum resources. Research by (Brakerski et al., 2021) and (Escolà-Farràs & Speelman, 2025) reveals that quantum strategies winning these games imply computational complexity separations. That is, Bell inequality violations do more than violate classical intuition—they demonstrate quantum computational supremacy in carefully defined tasks.

Methodology

In a quantum circuit, there exists a set of qubits and a following set of quantum gates applied to them. Qubits are the units of quantum information, like bits are in classical information, and exist in superpositions of the computational basis states $|0\rangle$ and $|1\rangle$. They are then written as $\alpha |0\rangle + \beta |1\rangle$ where α and β are complex amplitudes. Quantum gates then perform operations on these states, stemming into two branches, single-qubit gates such as Hadamard, Pauli and Phase gates and multi-qubit gates such as the CNOT. Quantum circuit diagrams can also be represented as diagrams with horizontal lines for qubits and boxes for gates, applied left to right, as seen in Figure 1.

Figure 1

Circuit Diagram for Preparing A GHZ State Using Hadamard and CNOT Gates. (Isaacdevlugt, 2023)



The GHZ State

The Greenberger-Horne-Zeilinger (GHZ) state is a multiparticle maximally entangled quantum state. No qubit is independent and full information about the state is only available when observing all qubits together. Measuring any one qubit instantly determines the state of the other two, showing the nonlocal correlation across the qubits. The GHZ state allows the maximal violation of a multipartite Bell inequality, allowing the opportunity to obtain the strongest violations.

To create the GHZ state (for n qubits), start with a qubit $|\psi\rangle = |0\rangle^{\otimes n}$. Next, applying a Hadamard gate to the first qubit (the Hadamard gate is a fundamental gate in quantum computing that creates a superposition in the qubit such that the qubit exists in both $|0\rangle$ and $|1\rangle$ with equal probability) the qubit becomes $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle^{\otimes n-1}$ which can then be simplified to $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes n} + |1>|0\rangle^{\otimes n-1})$ due to the distributive property of ket vectors. Now, applying $n-1$ CNOT gates where the control qubit is always 0 and the target qubit is i where $i \in \{1,2,\dots,n-1\}$, The end result is the GHZ state - $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes n} + |1^{\otimes n})$.

The CNOT gate keeps the control qubit system the same (the first state is kept the same here - $|0\rangle^{\otimes n}$) and flips the target qubit system ($|1\rangle \;\; |0\rangle^{\otimes n-1}$)

Bell Inequalities

Bell inequalities are mathematical expressions used to distinguish classical from quantum correlations. Essentially, they test whether the outcomes of measurements on entangled particles can be explained by local hidden variable theories. Bell inequalities are derived under two classical assumptions: Locality and Realism. Locality assumes that what happens in one system has no effect on another system, and realism assumes physical properties of an object are predetermined and are independent of observation. A Bell inequality has a classical and quantum bound where the breakage of the classical bound reflects the quantumness of the device. How close this violation can get to the quantum bound is then a reflection of how much more quantum one device is than another. A closer violation to the quantum bound reflects the lack of noise (interference) in the system, in which no noise allows the attainment of the quantum bound.

During this paper, 3 Bell Inequalities will be evaluated, the Mermin, Svetlichny and MABK. Their descriptions follow below -

Mermin Inequality

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The Mermin Inequality (Mermin, 1990) generally applies to three (or more) qubits, for the purposes of this paper, the 3-qubit version is used. There is a total of 6 measurements - $\widehat{A_1}$, $\widehat{A_2}$, $\widehat{B_1}$, $\widehat{B_2}$, $\widehat{C_1}$, $\widehat{C_2}$ that are combined into the Mermin expression below -

$$\widehat{M_3} = \widehat{A_1} \otimes \widehat{B_1} \otimes \widehat{C_2} + \widehat{A_1} \otimes \widehat{B_2} \otimes \widehat{C_1} + \widehat{A_2} \otimes \widehat{B_1} \otimes \widehat{C_1} - \widehat{A_2} \otimes \widehat{B_2} \otimes \widehat{C_2}$$

The hat symbol here ($\widehat{\ }$) represents quantum operators acting on a quantum state. These operators correspond to measurable physical quantities. For example, $\widehat{A_1}$ represents a particular measurement setting or observable associated with party A. Each observable such as $\widehat{A_1}$, $\widehat{A_2}$, $\widehat{B_1}$, $\widehat{B_2}$, $\widehat{C_1}$, $\widehat{C_2}$ and so on, corresponds to a specific measurement along a specific axis in the Bloch sphere representation. These are generally chosen to be spin measurements or Pauli operators like $\hat{X}, \hat{Y}$ or $\hat{Z}$ or more typically a linear combination of them, for example, $\widehat{A_1} = cos(\theta)\hat{X} + sin(\theta)\hat{Y}$ may be a spin measurement along a rotated axis. Each of these observables then returns a binary outcome either ± 1 and combinations of expectation values from these measurements are produced and used to produce the attained Mermin value. Although each quantum measurement has a result of +1 or –1, the expectation values used in Mermin-type inequalities are statistical averages over many runs. These averages are then continuous values between –1 and +1 and can take on decimal values. These expectation values are summed in the order a Bell inequality describes to attain a value for specific expressions. This is then applicable for the Svetlichny and MABK inequalities to be discussed below.

For this expression, the classical bound is 2 and the quantum bound is 4 (Alsina & Latorre, 2016). The violation of the Mermin inequality rules out fully local hidden variable theories which to breakdown means that the system exhibits quantum nonlocality.

Svetlichny Inequality

The Svetlichny Inequality (Svetlichny, 1987) is designed to specifically test for genuine tripartite nonlocality. Genuine tripartite locality means that the correlations require the input from all three qubits where no two-qubit entanglement model can reproduce results, the system also exhibits nonlocality that can't be broken down into simpler parts. The measurement settings remain the same as Mermin (A_1, A_2, B_1, B_2, C_1, C_2). The Svetlichny tests genuine tripartite nonlocality (this advances Mermin as Svetlichny rules out situations where two qubits may be entangled whilst one is locally correlated which Mermin includes) using the Svetlichny expression below.

$$\hat{S} = \widehat{A_1} \otimes \widehat{B_1} \otimes \widehat{C_1} + \widehat{A_1} \otimes \widehat{B_1} \otimes \widehat{C_2} + \widehat{A_1} \otimes \widehat{B_2} \otimes \widehat{C_1} - \widehat{A_1} \otimes \widehat{B_2} \otimes \widehat{C_2}$$

$$+ \ldots (terms\ with\ \widehat{A_2})$$

Where the classical bound is 4 and quantum bound can reach as high as $4\sqrt{2}$ (Svetlichny, 1987).

Mermin-Ardehali-Belinskii-Kylshko (MABK) Inequality

The MABK Inequality (Ardehali, 1992; Belinskii & Klyshko, 1993; Mermin, 1990) is a group of Bell inequalities that generalize the Bell and Mermin inequalities to an arbitrary number of qubits. Consequently, the classical and quantum bounds scale as the number of qubits increases. The advantage here is that it can test a range of qubits on quantum devices and get an idea for how the entire quantum device can perform. This is because when working with a fixed number of limited qubits (for example 3), the qubits that run may be of a higher quality compared to the other qubits, hence the benchmarking process is skewed. The observables remain the same as Mermin and Svetlichny, the exception clearly being an additional observable pair with each additional qubit (so for 4 qubit MABK, the observables are $\widehat{A_1}, \widehat{A_2}, \widehat{B_1}, \widehat{B_2}, \widehat{C_1}, \widehat{C_2}, \widehat{D_1}, \widehat{D_2}$).

For the 4-qubit MABK inequality, the operator is constructed recursively from lower-qubit MABK operators (Ardehali, 1992; Belinskii & Klyshko, 1993; Mermin, 1990). Specifically, the N-qubit MABK operator (Fan et al., 2023) M_N as

$$M_N = \frac{1}{2}[M_{N-1} \otimes (A_N + A'_N) + M'_{N-1} \otimes (A_N - A'_N)]$$

Where A_N and A'_N are the two observables for the N-th qubit, and M'_{N-1} is the same as M_{N-1} , but the measurement settings are swapped such that M'_N switches A_N and A_{N-1} with A'_N and A'_{N-1}, vice versa. The classical bound r is given as $2^{(n-1)/2}$ for odd n and $2^{n/2}$ for even n. The quantum bound is given as $2^{\frac{n+1}{2}}$ (Seevinck & Uffink, 2001). To allow the comparison across different qubit numbers, MABK values are then normalized between 0 and 1, where 0 is the classical bound and 1 is the quantum bound. The normalization thus allows a clear idea of how strongly the system violates classical constraints based on n qubits and serves as a useful comparative tool.

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Measuring quantum values

Measurement is the process of obtaining a classical output from a quantum state. Before measurement, the qubit is in a superposition of multiple states and so measuring it forces the qubit to randomly "pick" one state based on probabilities. For states like the GHZ state, this means collapsing each qubit's superposition to a definite outcome (0 or 1) in the computational basis (The Z-basis). The computational basis is the standard basis that quantum computers perform measurements on the axes of $|0\rangle$ and $|1\rangle$. Since the highest possible violations are of the Bell inequalities are desired, angle settings are used. Each measurement is rotated and then when measured in the computational basis, the qubits are then aligned in an ideal manner, the highest possible value is attainable. In addition, bell inequalities require the use of the XY plane, hence the use of rotation also serves to simulate measurement in the XY plane.

To measure a qubit in the GHZ state to compute the Mermin expression.

$$| \, GHZ\rangle = \frac{1}{\sqrt{2}}(| \, 000\rangle + | \, 111\rangle)$$

Then the observables for each qubit are defined here.

$$\boldsymbol{\widehat{O_i}(\theta_i) = cos(\theta_i)\sigma_x + sin(\theta_i)\sigma_y} \quad \text{where} \quad i \in \{A, B, C\}$$

Here σ_x is defined as a measurement along the x-axis and σ_y is defined as a measurement along the y-axis. These are two Pauli matrices used to represent measurements of a single qubit where

$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$. σ_x has eigenstates $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and σ_y has eigenstates $|+_y\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ and $|-_y\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$. In this context, eigenstates are specific qubit states that give a definite outcome when measured in σ_x or σ_y. Then when applying this to a qubit state, it checks to see how aligned the state is with the eigenstate, yielding a value between -1 and 1. If the qubit is fully in the state $|+\rangle$, a result of 1 is yielded and if the state is fully in $|-\rangle$, a result of -1 is yielded - when measured in σ_x. Similarly, this is applicable for σ_y.

To determine the rotation to map $\widehat{O_i}(\theta_i)$ to the computational basis, the function $U_i(\theta_i) = R_z(\theta_i) \cdot H$ is used.

Where:

$$R_z(-\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\theta} \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Then the local rotations are applied to the $|GHZ_3\rangle$ state

$$|\psi\rangle = (U(\theta_A) \otimes U(\theta_B) \otimes U(\theta_C))\,|GHZ_3\rangle$$

Where $|\psi\rangle$ is the rotated entangled state.

Since the rotations are applied, it is possible to measure the qubits on the computational basis.

Where the projectors are:

$P_0 = | 0\rangle \langle 0|$ and $P_1 = | 1\rangle \langle 1|$

These projectors represent quantum measurements in the computational basis. A projector is an operator that extracts the component of a quantum state aligned with a specific basis state. For example, applying P_0 onto the GHZ state $| \psi\rangle$, only keeps the part of $| \psi\rangle$ that lies in the direction of $| 0\rangle$.

And where the measurement probabilities are :

$p(xyz) = | \langle xyz | \psi\rangle |^2$ for $x, y, z \in \{0,1\}$

This formula then obtains the probability of the first qubit being state 'x', second qubit being state 'y' and third qubit being state 'z'.

We then map outcomes to either ±1 so that for each shot, let the computational-basis result be

$$(x, y, z) \in \{0, 1\}^3$$

Now to get the expectation value for a given setting.

With rotation angles $\theta_A^{(i)}, \theta_B^{(j)}, \theta_C^{(k)}$ where $i, j, k \in \{1,2\}$, the correlator is

$$\langle A_i B_j C_k \rangle = \sum_{x,y,z \in \{0,1\}} (-1)^{x+y+z} p_{ijk}(x,y,z)$$

Where $p_{ijk}(x,y,z) = | \langle xyz | \psi_{ijk} \rangle |^2$ and $| \psi_{ijk} \rangle = (U(\theta_A^{(i)}) \otimes U(\theta_B^{(j)}) \otimes U(\theta_C^{(k)})) | GHZ \rangle$

Noise (Interference) Sources

Even the best quantum systems today have imperfections. Interactions with the environment, faulty gate operations, and readout errors all cause noise which then alters the ideal evolutions of quantum states. In the context of Bell inequalities, noise reduces the strength of any quantum correlations and can inhibit attaining the highest possible violation and at times can barely even cause the classical bound to be broken. Thus, understanding the importance and characterizing noise is crucial when evaluating the Bell inequalities as quantum tests; for this study, five common noise channels were considered. Amplitude damping, bit flip, phase flip, phase damping and depolarizing noise.

Table 1

The Detailing of Each Tested Noise Source

Noise Type	Description	Effect on Bell Inequality / Entanglement
Amplitude Damping	Amplitude damping occurs when energy is lost to the environment, causing the qubit to relax from $\|1\rangle$ to $\|0\rangle$.	Amplitude damping destroys superposition and entanglement, especially in states like the GHZ state where it is maximally entangled, leading to sharp reductions in attained values as the amplitude damping strength increases.
Bit Flip	Bit flip noise occurs when there are random flips between $\|1\rangle \leftrightarrow \|0\rangle$ simulating classical bit errors.	As a result, when computing expressions, regardless of noise strength, the violation of the classical bound is still observed as bit flips do not destroy entanglement completely, hence, the obtained value still remains between the classical and quantum bound.
Phase Flip	Phase flip noise flips the phase of a qubit's state without altering its probability amplitudes.	It's a form of coherent error that acts on the relative phase between quantum states, especially affecting superpositions. Phase flip noise strongly affects measurements in the X and Y bases, hence whilst computing Bell inequality expressions, it can be expected that obtained values will be heavily affected.

Noise Type	Description	Effect on Bell Inequality / Entanglement
Phase Damping	Phase damping destroys quantum coherence between $\|0\rangle$ and $\|1\rangle$ without changing the energy.	As the phase damping strength increases, it destroys entanglement in the GHZ state, causing the qubits to become increasingly decoherent, hence it can be assumed that the calculated value for the expressions decreases as phase damping increases.
Depolarizing	Depolarizing noise randomly replaces a qubit's state with a completely mixed state with some probability.	It represents the scenario when a qubit "forgets" its information. As the depolarizing noise strength increases, there is an expectation that the calculated expression values will decrease drastically as depolarizing noise increases.

To test the effect of noise on the Bell inequalities, the PennyLane library and Python code was used. The PennyLane library was used to simulate the qubit system and various noise levels and then the Matplotlib library produced graphs showcasing the highest attained value as noise strength increased. The noise sources were easily obtainable through PennyLane's in-built imports. Finally, to ensure that every value obtained was the highest possible value the expression can attain, Powell Optimization was used.

To use this optimization tool, the program defined a cost function based on each Bell inequality. It then minimized the negatives of each expression to maximize the calculated values. As a result, the optimizer adjusts the $2n$ observables, where n is the number of qubits used, so that maximum quantum violation is attained. To avoid local optima, the code performs 'restarts' where, on each noise source and level, the optimizer runs a set number of times and the highest violation is recorded. See Appendix for Mermin, Svetlichny and MABK Programs.

Results / Conclusion

Figure 2

Best Mermin Value Attained Against Noise Strength

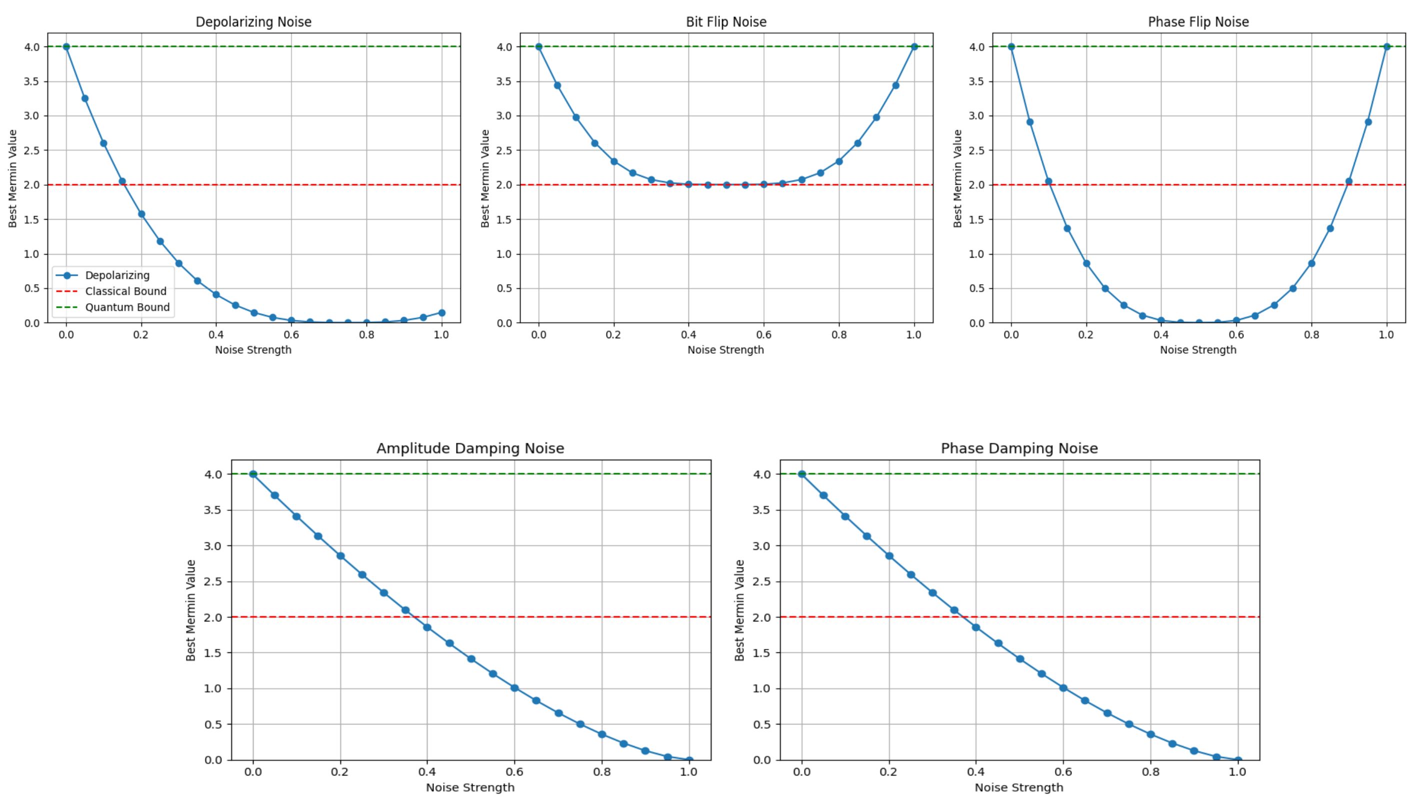

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Figure 3

Best Mermin Value Attained Against Noise Strength

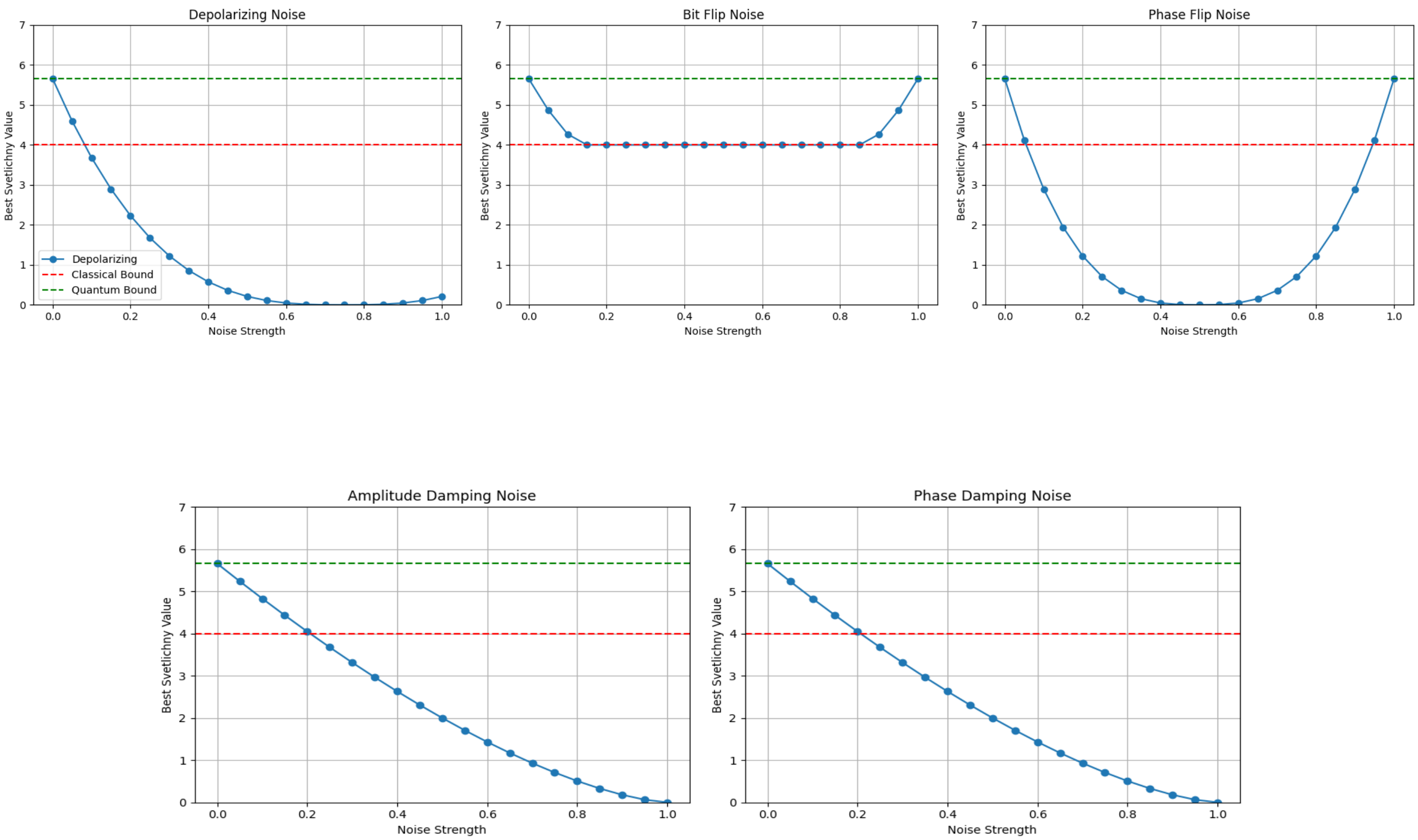
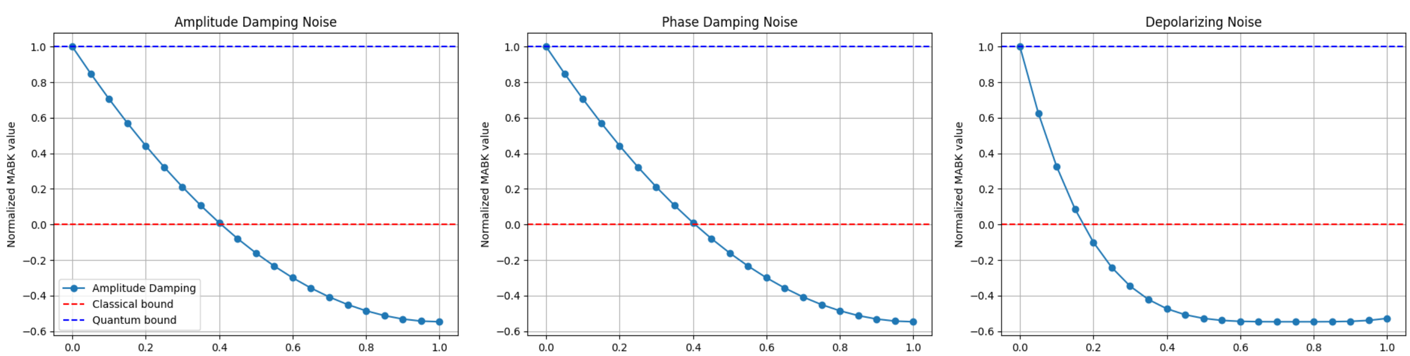


Figure 4

Best MABK For 4 Qubits Value Attained Against Noise Strength



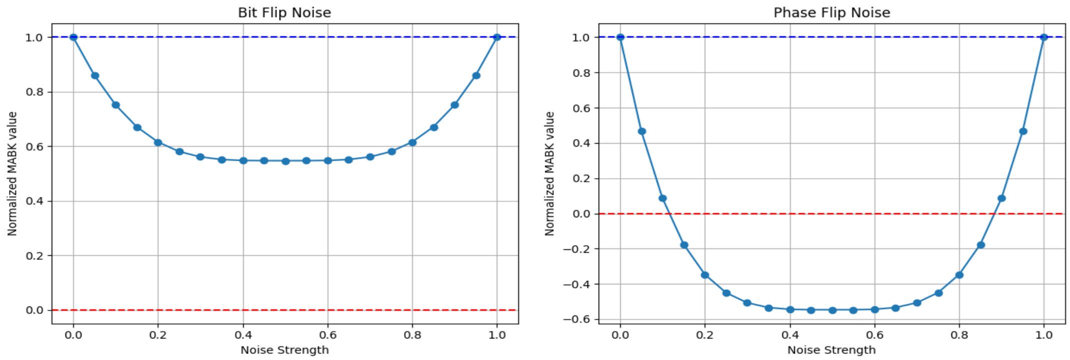
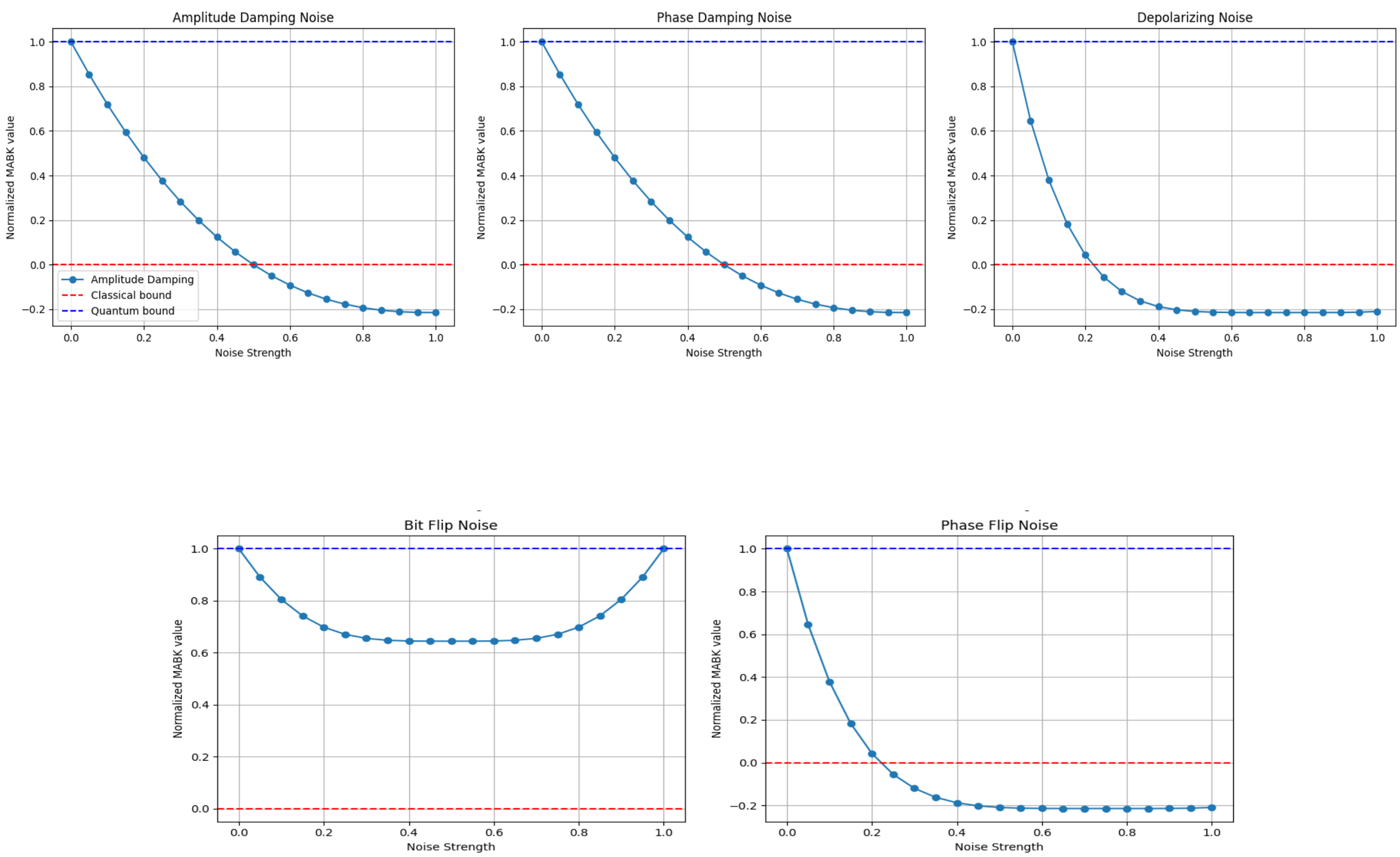


Figure 5.

Best MABK For 5 Qubits Value Attained Against Noise Strength



Looking at the results, it is evident that the impact of noise on Bell inequality violations depends on both the number of qubits involved and the type of noise channel. For the MABK inequality with 4 and 5 qubits, amplitude damping, phase damping, and depolarizing noise all show a steady decay of the normalized violation as noise strength increases. This thus shows that decoherence directly suppresses the entanglement responsible for the violations, eventually driving the values below the classical bound. However, bit flip and phase flip noise show more symmetric behavior, with violations dropping sharply in the mid-range of noise strength but recovering at the extremes, reflecting the fact that a complete flip can partially restore correlations.

Next, when comparing the Mermin and Svetlichny inequalities, a similar trend is observed: decoherence-type noise channels (amplitude damping, phase damping, depolarizing) steadily reduce violations, while flip-type noise introduces parabolic patterns. Additionally, it is important to note that the Mermin and Svetlichny inequalities both fall below their classical thresholds at moderate noise strengths, showing their sensitivity as benchmarks of quantumness. The fact that normalized values approach or drop under the classical threshold highlights the fragility of multipartite entanglement in realistic environments, emphasizing the practical challenge of preserving nonlocal correlations as qubit numbers increase. Overall, these results confirm that while current Bell inequalities can become valid tests of quantumness, their robustness is highly dependent on noise resilience—making noise mitigation a critical priority for future quantum computing.

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